Math 260: Linear Algebra
Chapter 5 & 6 Section 2 – Subspaces
Homework

1. (book section 5.1 #1)
In each case determine whether U is a subspace of $\mathbb{R}^3$. Support your answer.

a. $U = \{(1,\ s,\ t) \mid s \text{ and } t \text{ in } \mathbb{R}\}$.

b. $U = \{(0,\ s,\ t) \mid s \text{ and } t \text{ in } \mathbb{R}\}$.

c. $U = \{(r,\ s,\ t) \mid r,\ s,\ \text{ and } t \text{ in } \mathbb{R},\ -r+3s+2t=0\}$.

d. $U = \{(r,\ 3s,\ r-2) \mid r \text{ and } s \text{ in } \mathbb{R}\}$.

e. $U = \{(r,\ 0,\ s) \mid r^2+s^2=0,\ r \text{ and } s \text{ in } \mathbb{R}\}$.

f. $U = \{(2r,\ -s^2,\ t) \mid r,\ s,\ \text{ and } t \text{ in } \mathbb{R}\}$.

2. (book section 6.2 #1)

Which of the following are subspaces of $\mathbf{P}_3$? Support your answer.

a. $U = \{f(x) \mid f(x) \in \mathbf{P}_3,\ f(2) = 1\}$

b. $U = \{xg(x) \mid g(x) \in \mathbf{P}_2\}$

c. $U = \{xg(x) \mid g(x) \in \mathbf{P}_3\}$

d. $U = \{xg(x) + (1-x)h(x) \mid g(x) \text{ and } h(x) \in \mathbf{P}_2\}$

e. $U =$ The set of all polynomials in $\mathbf{P}_3$ with constant term 0

f. $U = \{f(x) \mid f(x) \in \mathbf{P}_3,\ \deg f(x) = 3\}$

3. (book section 6.2 #2)

Which of the following are subspaces of $\mathbf{M}_{22}$? Support your answer.

a. $U = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \middle| \, a,\ b,\ \text{and } c \text{ in } \mathbb{R} \right\}$

b. $U = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, a+b=c+d;\ a,\ b,\ c,\ d \text{ in } \mathbb{R} \right\}$

c. $U = \{A \mid A \in \mathbf{M}_{22},\ A = A^T\}$

d. $U = \{A \mid A \in \mathbf{M}_{22},\ AB = 0\}$, B a fixed 2×2 matrix

e. $U = \{A \mid A \in \mathbf{M}_{22},\ A^2 = A\}$

f. $U = \{A \mid A \in \mathbf{M}_{22},\ A \text{ is not invertible}\}$

g. $U = \{A \mid A \in \mathbf{M}_{22},\ BAC = CAB\}$, B and C fixed 2×2 matrices

4. (book section 6.2 #3)
Which of the following are subspaces of $\mathbf{F}[0,\ 1]$? Support your answer.

a. $U = \{f \mid f(0) = 0\}$

b. $U = \{f \mid f(0) = 1\}$

c. $U = \{f \mid f(0) = f(1)\}$

d. $U = \{f \mid f(x) \geq 0 \text{ for all } x \text{ in } [0,\ 1]\}$

e. $U = \{f \mid f(x) = f(y) \text{ for all } x \text{ and } y \text{ in } [0,\ 1]\}$

f. $U = \{f \mid f(x+y) = f(x) + f(y) \text{ for all } x \text{ and } y \text{ in } [0,\ 1]\}$

g. $U = \{f \mid f \text{ is integrable and } \int_0^1 f(x)dx = 0\}$